Evolution of beta

Vassilios Papathanakos*

March 23, 2018

1 Introduction

We study the behavior of beta for a variety of diversified portfolios. We are particularly interested in how the value of beta changes depending on:

- the time scales of returns used to measure it;
- the time periods over which it is measured, especially when they correspond to different market-risk regimes.

The first topic is the subject of Section 3, while the second topic is the subject of Section 4.

2 Data and methodology

2.1 Portfolios

In all cases, we consider the period 1992–2017, and the following seven portfolios (cf. Figure 1):

Portfolio	**Code**	r	σ	$\frac{r}{\sigma}$	β
MSCI All-Country World Index	M	7.45	15.26	0.489	1.000
MSCI All-Country World Diversity-Weighted Index	D	7.77	16.16	0.481	1.042
MSCI All-Country World Equal-Weighted Index	E	8.19	17.78	0.461	1.087
Intech Global All-Country Core strategy	C	10.20	15.42	0.662	0.997
Intech Global All-Country Low Volatility strategy	L	9.01	8.89	1.013	0.454
Intech Global All-Country Adaptive Volatility strategy	A	10.93	10.37	1.054	0.571
Intech Global All-Country Hybrid Volatility Select	H	10.50	9.81	1.070	0.522

Table 1: The portfolios studied in this report, including some performance statistics over the entire period. All returns are logarithmic; also, these betas are computed as the regression coefficient of the portfolios' monthly, arithmetic, absolute returns with respect to the capitalization-weighted benchmark. Finally, the diversity-weighted index corresponds to the diversity exponent $p = \frac{1}{2}$.

The first three portfolios (`M`, `D`, and `E`) are simulated in the absence of transaction costs; the last four portfolios (`C`, `L`, `A`, and `H`) are simulated in the presence of transaction costs[1].

*Intech; contact at VPapathanakos@intechinvestments.com

[1] When included, transaction costs are assumed to be 40bps and 80bps per unit trading distance for developed- and emerging-markets equity respectively.

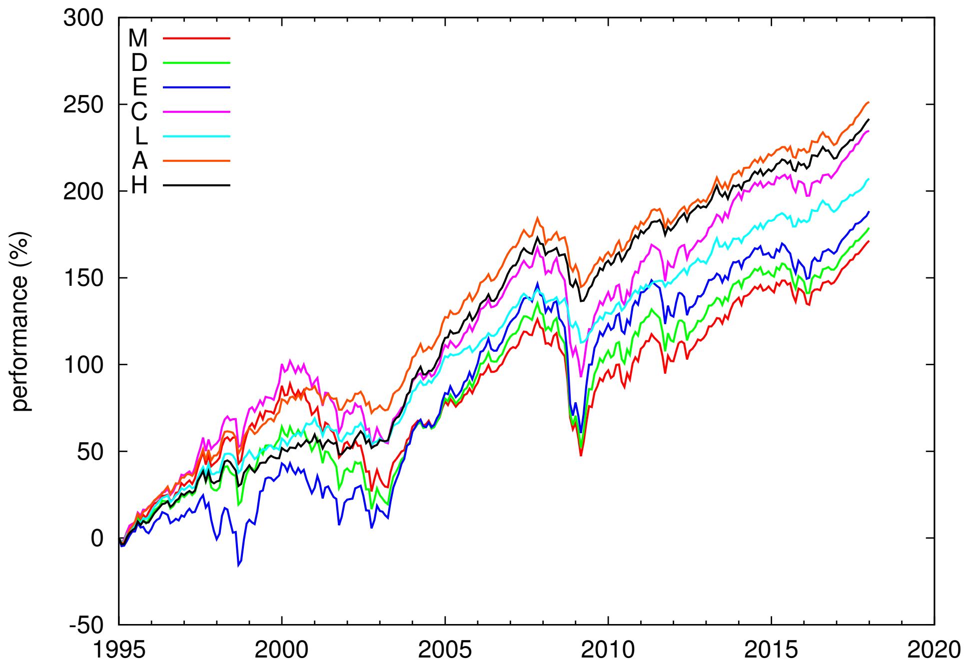

Figure 1: Cumulative performance of the five portfolios studied in this report.

2.2 Beta computation

The beta is computed below in a relatively straightforward manner:

- For individual stocks, it is the regression coefficient of their returns with respect to the capitalization-weighted benchmark's returns. These returns are sampled every D days, and over a period lasting P days:

$$\beta_s = \frac{\sum_{i=1}^{\lfloor P/D \rfloor} R_b\left(t_0 + D\, i\right) R_s\left(t_0 + D\, i\right) - \frac{1}{\lfloor P/D \rfloor} \sum_{i=1}^{\lfloor P/D \rfloor} R_b\left(t_0 + D\, i\right) \sum_{i=1}^{\lfloor P/D \rfloor} R_s\left(t_0 + D\, i\right)}{\sum_{i=1}^{\lfloor P/D \rfloor} R_b^2\left(t_0 + D\, i\right) - \frac{1}{\lfloor P/D \rfloor} \left(\sum_{i=1}^{\lfloor P/D \rfloor} R_b\left(t_0 + D\, i\right) \right)^2}, \quad (1)$$

  where R_b and R_s are respectively the benchmark's and stock's arithmetic, absolute, USD-denominated returns over the period starting on the date in parentheses.

- For diversified portfolios, it is the portfolio-weighted average of the betas of their constituents:

$$\beta_p = \frac{\sum_s p_s\, \beta_s}{\sum_s p_s}, \quad (2)$$

  where the p's are the portfolio's weights.

Figure 2 shows the evolution of beta over time in the simplest case (daily sampling over the past year). The beta of the market (portfolio `M`) is nominally one; in practice, it can vary depending on the reconstitution of the index, and the evolution of the market weights. The deviation of the market's beta from one furnishes a rough lower-bound on the accuracy of the beta estimation. Portfolio `C`, which attempts to control active risk, maintains a beta close to one; also, it is lower than the market's beta at almost all times. Portfolios `L`, `A`, and `H`, which attempt to control absolute risk, exhibit a consistently lower beta, which varies in a broad range (0.21–0.67, 0.24–0.91, and 0.14–0.79 respectively). Finally, size-exposed, unmanaged portfolios `D` and `E` exhibit an unconstrained beta behavior.

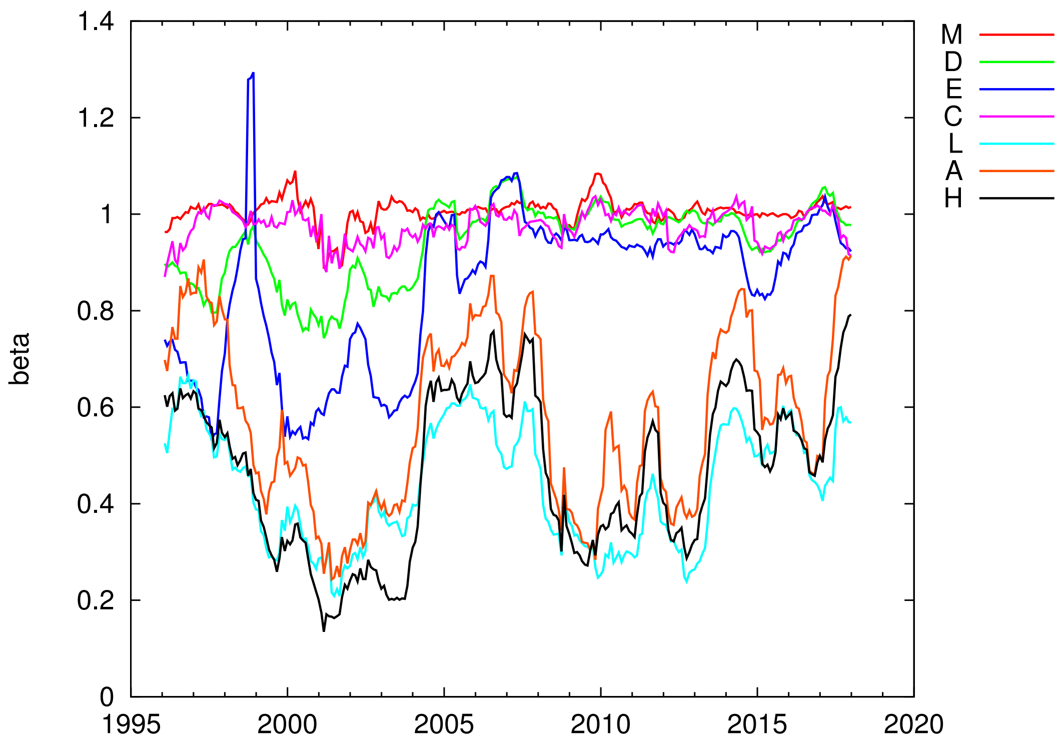

Figure 2: Evolution of beta for daily sampling over the past year ($D = 1$, $P = 260$).

3 Time scales

3.1 Sampling interval

In order to explore the effect of time scales, we first keep the look-back period fixed at one year, and vary the sampling interval. This furnishes one measure of the impact of serial correlation in the portfolio's performance. As shown in Figure 3, the average beta is stable for the market (which has a nominal beta of one), and for the optimized portfolios. However, the betas of the diversity-weighed portfolio `D` and the equal-weighted portfolio `E` are much more sensitive to the sampling period; in fact, they transition to beta greater than one at around $D = 10$.

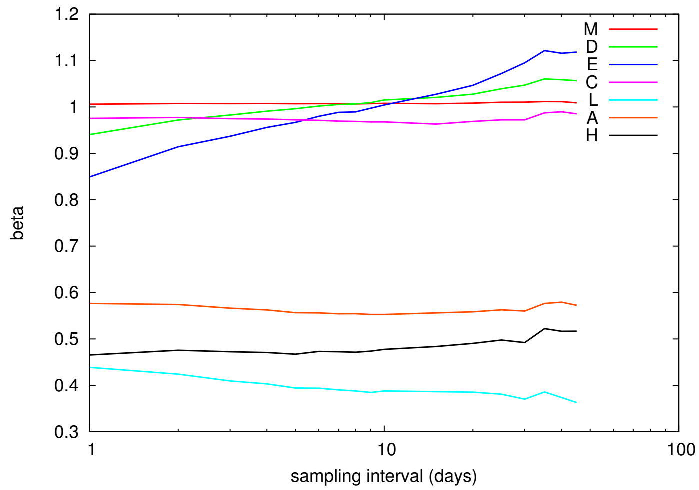

Figure 3: Average beta over simulation period depending on sampling interval D, at a fixed one-year look-back period ($P = 260$).

3.2 Look-back period

Another way to explore the effect of time scale is to keep the sampling interval fixed at one day, and vary the look-back period. As shown in Figure 4, the average beta continues to be stable for the market (which has a nominal beta of one). However, for optimized portfolios, the beta tends to decrease as the look-back period increases. Finally, the betas of the unmanaged diversity-weighted portfolio `D` and equal-weighted portfolio `E` stay much more stable independently of the look-back period.

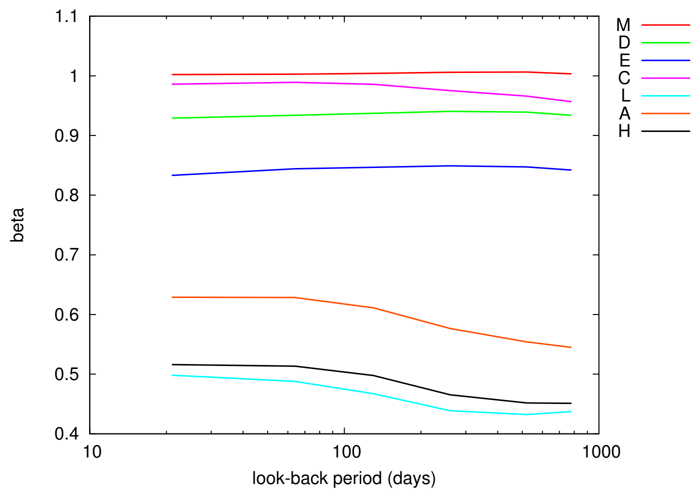

Figure 4: Average beta over simulation period depending on fixed look-back period P, at a fixed one-year sampling interval of one day ($D = 1$).

4 Market regimes

We divide the period 1992–2017 into six regimes of higher volatility, based on whether the rolling 12-month annualized monthly volatility of the market portfolio is higher than the full-period average or not. This is an *ad hoc* criterion, but it illustrates some basic properties of higher volatility environments. We are particularly interested in how the volatility, both absolute (as in standard deviation) and systematic (as in beta) of a portfolio evolves in reaction to market conditions.

In each of the following figures, we plot the beta and the absolute volatity of each portfolio using lines of the same color, but with different thickness. In order to have quantities of comparable magnitude, we normalize the beta and the rolling volatility by dividing them with appropriate long-term averages.

4.1 Unmanaged, size-exposed portfolios

As shown in Figure 5, the unmanaged portfolios experience overall risk at about the same level as the market, but the contribution of systematic equity risk (quantified by beta) varies considerably.

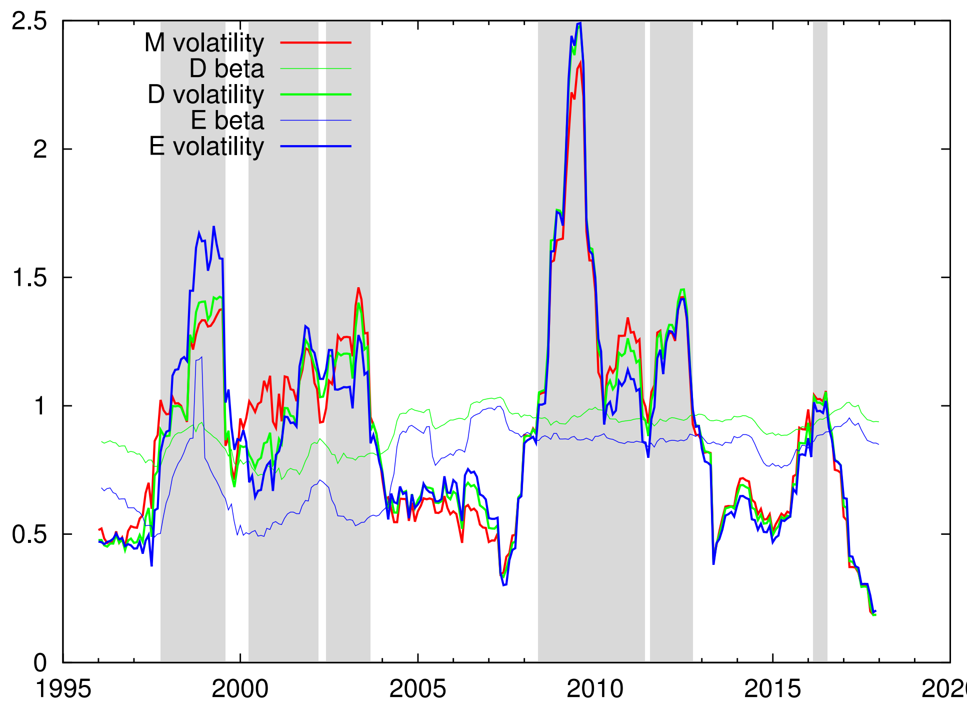

Figure 5: Rolling 12-month volatility of `M` (normalized with respect to 15.26%), rolling beta of `D` (normalized with respect to 1.042), rolling 12-month volatility of `D` (normalized with respect to 16.16%), rolling beta of `E` (normalized with respect to 1.087), and rolling 12-month volatility of `E` (normalized with respect to 17.78%).

4.2 Relative-risk portfolio

In Figure 6, we see an example of a relative-risk strategy: its volatility follows closely that of the market, and its beta is largely unaffected by the market regime.

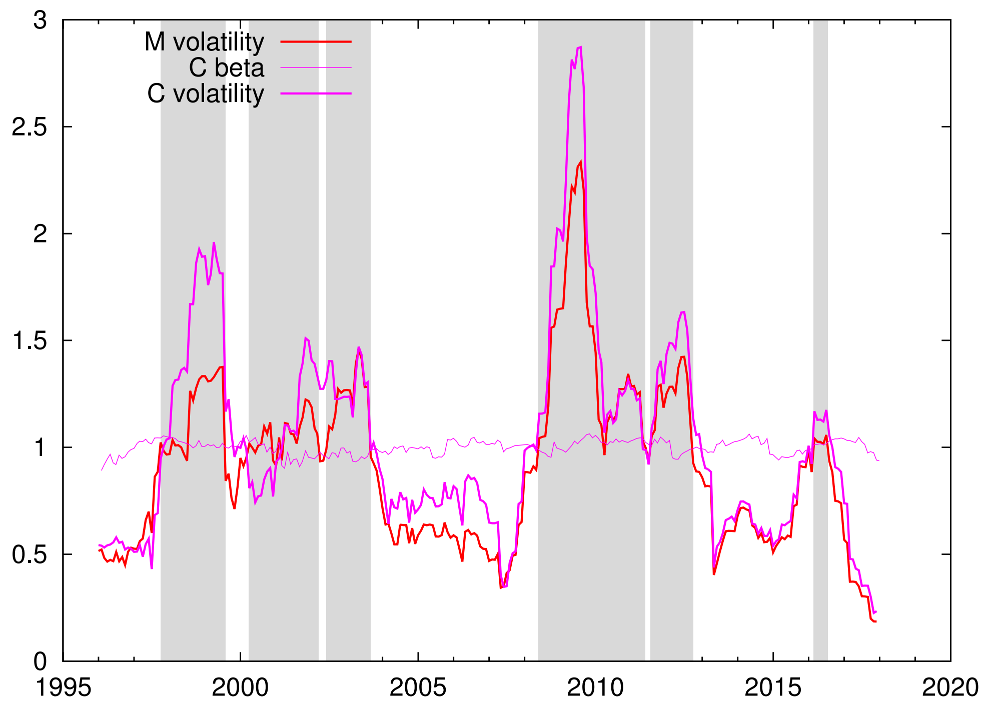

Figure 6: Rolling 12-month volatility of `M` (normalized with respect to 15.26%), rolling beta of `A` (normalized with respect to 0.9753), and rolling 12-month volatility of `A` (normalized with respect to 15.42%).

4.3 Absolute-risk portfolios

In Figure 7, we see three examples of an absolute-risk strategy: their total volatilities exhibit a more narrow range than the market (highs are considerably suppressed, lows are slightly uplifted compared to the benchmark):

Portfolio	**Minimum rolling volatility**	**Maximum rolling volatility**
`M`	2.8% (Dec 2017)	35.6% (Aug 2009)
`L`	3.5% (Dec 2017)	15.7% (Jul 2009)
`A`	3.3% (Dec 2017)	17.0% (Aug 2009)
`H`	3.1% (Dec 2017)	17.4% (Aug 2009)

Table 2: Ranges of the 36-month rolling volatility.

Note that the historical minimum for the rolling volatility occurs at the end of the simulation period (December 2017); the previous minimim occurred in May 2007, on the eve of the Global

Financial Crisis[2].

It is also clear that the betas of all three absolute-risk strategies are anticorrelated to the market volatility. This is achieved partly through employing a diversified portfolio (that will tend to absorb market shocks much better than the capitalization-weighted index), and partly through adapting to the market levels of risk. The relative significance of these two mechanisms depends on how long a regime lasts, and how abrupt the transition from the previous regime.

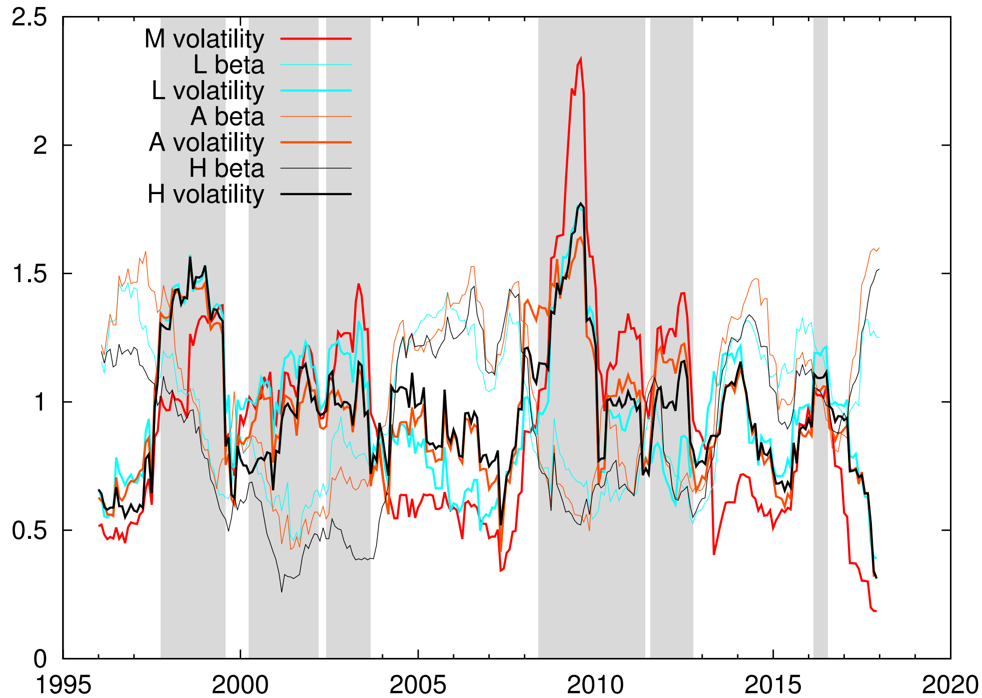

Figure 7: Rolling 12-month volatility of `M` (normalized with respect to 15.26%),rolling beta of `L` (normalized with respect to 0.454), rolling 12-month volatility of `L` (normalized with respect to 8.89%), rolling beta of `A` (normalized with respect to 0.571), rolling 12-month volatility of `A` (normalized with respect to 10.37%), rolling beta of `H` (normalized with respect to 0.522), and rolling 12-month volatility of `H` (normalized with respect to 9.81%).

5 Acknowledgment

We would like to thank James McHugh, Richard Yasenchak, CFA, Valerie Azuelos, Kevin Armstrong, CFA, Adrian Banner, PhD, and Lin Zhao, PhD, for helpful discussions. Also, we would like to thank an unnamed Intech client for a valuable discussion that prompted this analysis.

[2] For more details, please consult the Intech Equity Market Stress Monitor.

A Equal-weighted portfolio beta

A.1 Introduction

The behavior of the diversity-weighted and equal-weighted portfolio's beta over different sampling intervals in Figure 3 is quite striking. The performance of a diversity-weighted portfolio relative to the cap-weighted benchmark can be understood approximately as the dilution of the performance of the equal-weighted portfolio relative to the benchmark:

$$r_d(t) - r_b(t) \simeq p\left(r_e(t) - r_b(t)\right) , \tag{3}$$

where r_b, r_e, and r_d are the logarithmic, absolute returns of the benchmark, the equal-weighted portfolio, and the diversity-weighted portfolio respectively (for the example in Figure 3, $p = \frac{1}{2}$.). This means that it is sufficient to understand the behavior of the equal-weighted portfolio alone.

A plausible explanation is that small-cap stocks exhibit a high amount of volatility, both systematic (measured through beta) and idiosyncratic, and that the idiosyncratic volatility exhibits strong negative correlation. These assumptions imply that, over the short term, the idiosyncratic volatility suppresses the measured beta. However, when sampling at longer time scales, the idiosyncratic volatility no longer dominates, resulting in higher beta. In this section, we evaluate the merits of this explanation.

A.2 Computation of beta

In order to evaluate the limitations of the main methodology, we first consider an alternate computation for the equal-weighted beta. Instead of the approach of Section 2.2, where we first compute the stocks' betas and then portfolio-weight them, we compute the portfolio beta directly (cf. Figure 8).

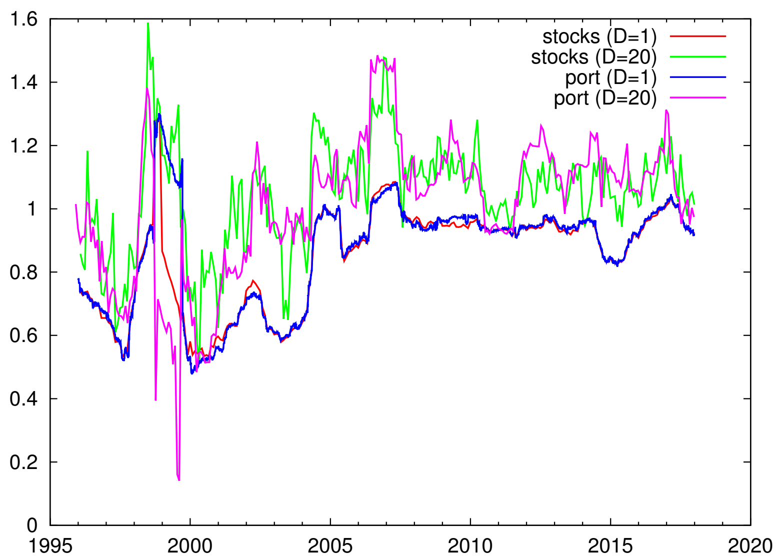

Figure 8: Beta of the equal-weighted portfolio computed through estimating the stocks' betas (red: $D = 1$; green: $D = 20$), and directly (blue: $D = 1$; violet: $D = 20$), at a fixed one-year look-back period.

Part of the difference between the two approaches is that, when estimating the stocks' betas first, reconstitution tends to 'clean up' the past return history. Conversely, extreme returns will continue to appear as outliers in the regression for the portfolio returns in the second methodology. Consider for example the anomalous behavior in the late 1990's, which is due to the Asian Financial crisis, especially the behavior of Russian stocks that exhibited extreme negative returns. When using stocks' betas (red line), the spike in beta disappears after the index reconstitution in December 1998. On the contrary, when using the overall portoflio returns (blue line), the spike in beta disappears months later (in September 1999), after the rolling 12-period window no longer includes the extreme returns.

A.3 Distribution of stocks' betas

We next turn our attention to the distribution of betas across the stocks in the investable universe. Figure 9 shows how the histogram of betas changes shape as the sampling interval D lengthens. The median and the mode of the distribution increase to one, while the breath of the distribution expands.

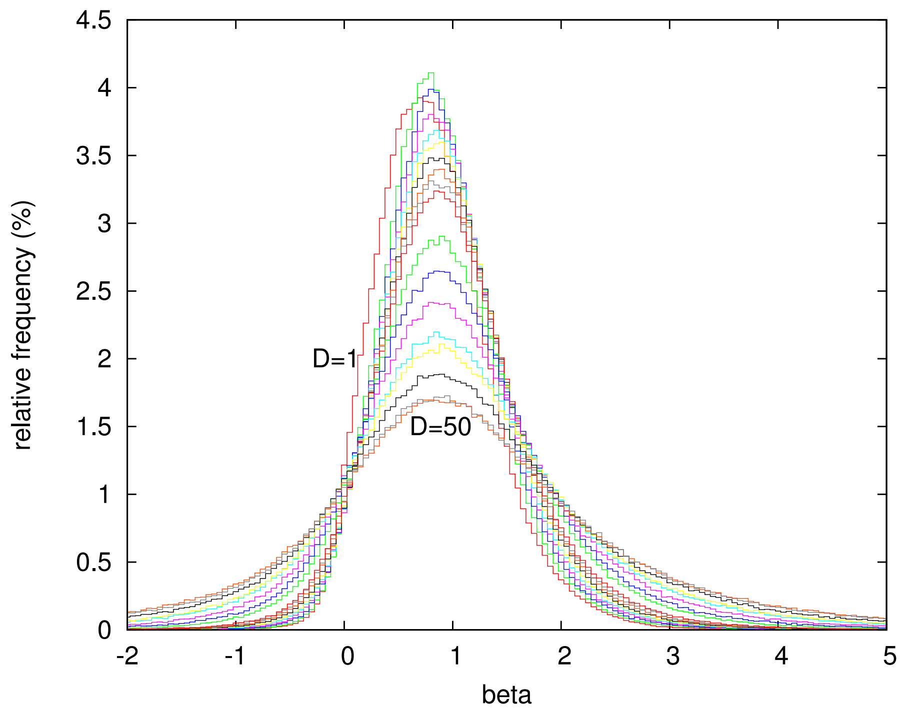

Figure 9: Histogram of the stocks' betas for sampling interval of $D = 1, 2, \ldots, 10, 15, \ldots, 50$, at a fixed one-year look-back period ($P = 260$).

This plot is supportive of the explanation, but is not completely conclusive, especially since further analysis appears to indicate that both large- and small-cap stocks behave in a similar manner. Further analysis is required to resolve this open question.

Simulations Disclaimer

Simulations have been compiled solely by Intech and have not been independently verified. Intech’s simulations are produced with the benefit of hindsight by applying its mathematical optimization process to historical data, and unlike traditional simulations, do not involve fundamental estimates. Simulations are hypothetical, not real, and are presented to potentially allow investors to understand and evaluate Intech’s investment process by seeing how a product may have performed during certain time periods. Simulations do not reflect results or risks associated with actual trading of an account, and there is no guarantee that an actual account would have achieved similar results. In no circumstances should simulated results be regarded as any representation, guarantee, assumption, or prediction of future performance, or that investors will be able to avoid losses. Investing involves risk, including fluctuation in value, the possible loss of principal and total loss of investment. Past performance of simulated data is no guarantee of future results.

Simulations do not reflect numerous other material economic, market, and implementation factors that may have impacted Intech’s trading or decision-making in the actual management of an account and cannot be fully accounted for in the preparation of simulations, all of which can adversely affect actual results. The simulated results include the reinvestment of dividends, interest, and capital gains, but do not reflect the deduction of investment advisory fees or trading costs unless otherwise noted. Thus, simulated results will be reduced by advisory fees, trading costs, and any other expenses that may be incurred in the management of an account, which will materially lower results over time.